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Direct CP-violation as a test of quantum mechanics

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Abstract. Direct CP-violating effects in the neutral kaon system result in violations of certain Bell-like inequalities. The new experimental results on the determination of the phenomenological parameter ε' allow to dismiss a large class of "hidden variable" alternatives to quantum mechanics.

1 Introduction

The entangled character of neutral kaon pairs produced at ϕ-factories offers the possibility of meaningful tests of Bell's locality [1, 2] in a broad class of hidden-variable extensions of ordinary quantum mechanics. This may be achieved in essentially two different ways, either by exploiting the correlations at different times involving strangeness oscillations, [3-9] or by identifying different kaon states via their decay products, with or without using regeneration methods.[10-14]

A different, more general point of view is however possible; as suggested in [15], a clean confirmation of quantum mechanics against local hidden-variable theories, rather than by correlation measures, might come from an accurate determination of the phenomenological quantity ε' by whatever means obtained. The complex parameter ε' characterizes direct CP-violation in the neutral kaon system and is predicted to be non-zero by the Standard Model.[16-18]

Indeed, as discussed below, inequalities of Clauser-Horne type [19, 20] can be derived for hidden-variable theories that reproduce neutral kaon's phenomenology and comply with the hypothesis of stochastic independence of neutral kaon decays. Then, a specific Bell's inequality is showed to be violated if ε' is measured to be non-zero.

A preliminary result on the determination of the phenomenological quantity $\mathcal{R}e(\varepsilon'/\varepsilon)$ has been recently announced by two experimental collaborations [21, 22] allowing an estimate of the parameter ε'. We shall explicitly show that, as a byproduct, these measurements represent the first experimental evidence of a violation of Bell's locality occurring in a subnuclear system, without requiring correlation measures. We will make clear that the violation comes about because of an internal inconsistency of a large class of hidden-variable theories that pretend to reproduce standard kaon phenomenology in the presence of direct CP-violation.

2 The neutral kaon system

The standard effective description of neutral kaons makes use of a two dimensional Hilbert space.[16] A useful orthonormal basis in this space is given by the CP-eigenstates

$$|K_1\rangle = \frac{|K^0\rangle + |\overline{K^0}\rangle}{\sqrt{2}} , \qquad |K_2\rangle = \frac{|K^0\rangle - |\overline{K^0}\rangle}{\sqrt{2}} , \quad (2.1)$$

where $|K^0\rangle$, $|\overline{K^0}\rangle$ are the strangeness eigenstates. The time-evolution and decay of neutral kaons is described by the nonhermitian phenomenological hamiltonian $H_{\rm eff} = M - i\Gamma/2$, with M and Γ the positive 2×2 mass and decay matrices. A kaon, initially in a state $|K\rangle$, will evolve up to its proper-time τ into a state $|K(\tau)\rangle = \exp(-i\tau H_{\rm eff})|K\rangle$.

As CP-invariance is not preserved in kaon time-evolution, the eigenstates of the hamiltonian $H_{\rm eff}$ are two non-orthogonal admixtures of the states (2.1), which, in the $|K_1\rangle$, $|K_2\rangle$ basis, are given by

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon_S|^2}} \begin{pmatrix} 1 \\ \epsilon_S \end{pmatrix} ,$$
$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon_L|^2}} \begin{pmatrix} \epsilon_L \\ 1 \end{pmatrix} . \quad (2.2)$$

The states $|K_S\rangle$, $|K_L\rangle$ correspond to the eigenvalues $\lambda_{S,L} \equiv m_{S,L} - i\gamma_{S,L}/2$, where m_S, m_L and γ_S, γ_L are the masses, and widths of the physical short and long-lived kaons. The complex quantities $\epsilon_{S,L}$ signal indirect CP and CPT-violating effects.

The decay-rate, or probability per unit time that a kaon in a generic state $|K\rangle$ decays into a final state f can be written as

$$\Gamma(K \to f) = \int {\rm d}\Omega_f \ |\mathcal{A}(K \to f)|^2 , \quad (2.3)$$

where $\mathcal{A}(K \to f)$ is the Lorentz-invariant amplitude which depends in general on the so-called Dalitz variables, while $d\Omega_f$ represents the corresponding phase-space measure.

Because of the linearity of the decay-amplitudes with respect to the kaon states, the effective description allows one to associate the decay into a specific final state f with a 2×2 positive operator $\widetilde{\mathcal{O}}_f$. In fact, using the decay amplitudes of the CP-eigenstates $|K_1\rangle$ and $|K_2\rangle$ into f, one constructs the matrix

$$\widetilde{\mathcal{O}}_f \equiv |\mathcal{A}(K_1 \to f)|^2 \begin{pmatrix} 1 & r_f \\ r_f^* & |r_f|^2 \end{pmatrix} , \quad r_f = \frac{\mathcal{A}(K_2 \to f)}{\mathcal{A}(K_1 \to f)} . \tag{2.4}$$

The parameter r_f can be expressed in terms of $\epsilon_{S,L}$ and the corresponding ratio of amplitudes for the decays of the states $|K_L\rangle$, $|K_S\rangle$ into f; these quantities are accessible to the experiment.

In this way, the decay-rate (2.3) can equivalently be written as the following mean-value with respect to the kaon state $|K\rangle$:

$$\Gamma(K \to f) = \int d\Omega_f \ \langle K|\widetilde{\mathcal{O}}_f|K\rangle = \langle K|\Big(\int d\Omega_f \, \widetilde{\mathcal{O}}_f\Big)|K\rangle . \tag{2.5}$$

Since $\widetilde{\mathcal{O}}_f$ has zero determinant, it is proportional to a projector

$$\mathcal{O}_f = \frac{1}{1+|r_f|^2} \begin{pmatrix} 1 & r_f \\ r_f^* & |r_f|^2 \end{pmatrix} = |K_f\rangle\langle K_f| , \tag{2.6}$$

where

$$|K_f\rangle = \frac{1}{\sqrt{1+|r_f|^2}} \begin{pmatrix} 1 \\ r_f^* \end{pmatrix} . \tag{2.7}$$

Further, to any specific final state f one associates an orthonormal basis in the effective Hilbert space. In fact, $\mathcal{O}_f^\perp \equiv 1 - \mathcal{O}_f$ projects onto the state $|K_f^\perp\rangle$,

$$\mathcal{O}_f^\perp = |K_f^\perp\rangle\langle K_f^\perp| , \qquad |K_f^\perp\rangle = \frac{1}{\sqrt{1+|r_f|^2|}} \begin{pmatrix} r_f \\ -1 \end{pmatrix} , \tag{2.8}$$

such that $\langle K_f|K_f^\perp\rangle = 0$; note that $|K_f^\perp\rangle$ cannot decay into f:

$$\mathcal{A}(K_f^\perp \to f) \propto r_f \mathcal{A}(K_1 \to f) - \mathcal{A}(K_2 \to f) = 0 . \tag{2.9}$$

Since kaons are spinless, in the case of two-body final states f the decay amplitudes are constant. Therefore,

$$\Gamma(K \to f) = \langle K|\widetilde{\mathcal{O}}_f|K\rangle \, \Omega_f \equiv |\langle K|K_f\rangle|^2 \, \mathrm{Tr}(\widetilde{\mathcal{O}}_f) \, \Omega_f , \tag{2.10}$$

where Ω_f is just a constant phase-space contribution. Let us point out that the probability $|\langle K|K_f\rangle|^2$ that a kaon in the state $|K\rangle$ be in the state $|K_f\rangle$ is directly related to a decay-rate. It can actually be measured; indeed, the factor $\mathrm{Tr}\,(\widetilde{\mathcal{O}}_f)$ in (2.10) can be extracted from the following branching ratios [10]

$$BR(K_{S,L} \to f) = \mathrm{Tr}\ (\widetilde{\mathcal{O}}_f) \, \frac{|\langle K_{S,L}|K_f\rangle|^2}{\gamma_{S,L}} \, \Omega_f , \tag{2.11}$$

where the scalar products $\langle K_{S,L}|K_f\rangle$ are fixed by equations (2.2) and (2.7) in terms of the parameter r_f in (2.4).

When the final state f comprises more than two particles, the decay amplitudes are not constant and depend on the appropriate Dalitz variables. One can still define states $|K_f\rangle$, but only for fixed kinematical configurations. Then, the operator $\int d\Omega_f \, \widetilde{\mathcal{O}}_f$ is clearly not proportional to a projector, but rather to a density matrix:

$$\rho_f = \frac{\int d\Omega_f \, \widetilde{\mathcal{O}}_f}{\mathrm{Tr}\Big[\int d\Omega_f \, \widetilde{\mathcal{O}}_f\Big]} . \tag{2.12}$$

In such cases, the probability that the kaon state $|K\rangle$ be in the mixture ρ_f is still proportional to the decay-rate of $|K\rangle$ into f, but the proportionality factor, *i.e.* the denominator in (2.12), cannot be easily related to measurable quantities.

In the sequel, we will deal with two-pion, $f_{00} = \pi^0\pi^0$, $f_{+-} = \pi^+\pi^-$, and semileptonic, $f_{\ell+} = \ell^+\pi^-\nu$, final states. In the basis $|K_1\rangle$, $|K_2\rangle$, a convenient parametrization of the corresponding $\widetilde{\mathcal{O}}_f$ in terms of phenomenologically measurable parameters can be given (for a discussion and more details, see [23, 24]). Indeed, following the previous considerations, to the pion final states we associate the projectors

$$\begin{aligned} \mathcal{O}_{+-} &= |K_{+-}\rangle\langle K_{+-}| \\ &= \frac{1}{1+|r_{+-}|^2} \begin{pmatrix} 1 & r_{+-} \\ r_{+-}^* & |r_{+-}|^2 \end{pmatrix} , \end{aligned} \tag{2.13a}$$

$$\begin{aligned} \mathcal{O}_{00} &= |K_{00}\rangle\langle K_{00}| \\ &= \frac{1}{1+|r_{00}|^2} \begin{pmatrix} 1 & r_{00} \\ r_{00}^* & |r_{00}|^2 \end{pmatrix} , \end{aligned} \tag{2.13b}$$

where the small parameters r_{+-} and r_{00} can be written as

$$r_{+-} = \varepsilon - \epsilon_L + \varepsilon' , \qquad r_{00} = \varepsilon - \epsilon_L - 2\varepsilon' ; \tag{2.14}$$

the two phenomenological quantities ε and ε' signal CP and CPT-violating effects that occur directly in the decay amplitudes and are used to parametrize the following constant amplitude ratios [16-18]

$$\begin{aligned} \eta_{+-} &= \frac{\mathcal{A}(K_L \to \pi^+\pi^-)}{\mathcal{A}(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon' , \\ \eta_{00} &= \frac{\mathcal{A}(K_L \to \pi^0\pi^0)}{\mathcal{A}(K_S \to \pi^0\pi^0)} = \varepsilon - 2\varepsilon' . \end{aligned} \tag{2.15}$$

Concerning the semileptonic decays, three body final states, the corresponding projector operators $\mathcal{O}_\ell$ depend in general on suitable Dalitz variables. Nevertheless, one usually takes the approximation of neglecting the lepton mass;[16] in this case, one can show that $\mathcal{O}_\ell$ is constant so that a state $|K_\ell\rangle$ can be defined and the corresponding probabilities $|\langle K|K_\ell\rangle|^2$ can be related to measurable quantities, as explained before. However, this approximation is not necessary if one assumes, as we will do, the validity of the socalled $\Delta S = \Delta Q$ rule;[16, 17] in this case only

K^0 and not $\overline{K^0}$ can decay into $\ell^+\pi^-\nu$. This allows us to associate to the final state $\ell^+\pi^-\nu$ the constant projector

$$\mathcal{O}_{\ell+} = |K_{\ell+}\rangle\langle K_{\ell+}| \equiv |K^0\rangle\langle K^0| = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} . \quad (2.16)$$

In the following, we deal with the probabilities $\langle K(\tau)|\mathcal{O}_{+-}|K(\tau)\rangle$, $\langle K(\tau)|\mathcal{O}_{00}|K(\tau)\rangle$ and $\langle K(\tau)|\mathcal{O}_{\ell+}|K(\tau)\rangle$, that a kaon evolved up to time τ into a state $|K(\tau)\rangle$ be in one of the states $|K_{+-}\rangle$, $|K_{00}\rangle$ and $|K_{\ell+}\rangle$; as shown above, all these probabilities are expressed in terms of experimentally accessible quantities.

A typical situation in which neutral kaon physics can be studied is offered by the socalled ϕ-factories. In such setups, ϕ-mesons are copiously produced, which mainly decay into couple of kaons. In the case of neutral kaons, because of symmetry reasons, the resulting state is entangled in a way that resembles the singlet state of two spin 1/2 particles:

$$|\Psi\rangle = N\Big(|K_S\rangle \otimes |K_L\rangle - |K_L\rangle \otimes |K_S\rangle\Big) , \quad (2.17)$$

where N is a normalization constant. The two kaons fly apart with opposite momentum (in the ϕ rest frame) and evolve each up to its proper time τ_i, $i = 1, 2$, according to the effective hamiltonian $H_{\rm eff}$:

$$|\Psi\rangle \mapsto |\Psi(\tau_1, \tau_2)\rangle \equiv \Big(e^{-iH_{\rm eff}\tau_1} \otimes e^{-iH_{\rm eff}\tau_2}\Big)|\Psi\rangle . \quad (2.18)$$

Let us observe that, setting $\tau_1 = \tau_2 = \tau$, the entanglement is preserved by the time-evolution:

$$|\Psi(\tau)\rangle \equiv \Big(e^{-iH\tau} \otimes e^{-iH\tau}\Big)|\Psi\rangle = e^{-i\Delta m\tau - \gamma\tau}|\Psi\rangle , \quad (2.19)$$

where $\Delta m = m_L - m_S$ and $\gamma = (\gamma_S + \gamma_L)/2$ mediates the exponential damping due to the system instability. The similarity with the standard Einstein-Poldosky-Rosen setting is apparent; this allows various tests of quantum mechanics to be performed.[1, 3-9, 25]

3 Bell-like inequalities with neutral kaons

Using (2.18) or (2.19), one computes probabilities related to double-decays and therefore tests of local hidden-variable theories in the neutral kaon context can be discussed.

Because of (2.19), the probabilities $\mathcal{P}_\tau(K_a, K_b)$ that one kaon be in a state $|K_a\rangle$ and the other in a state $|K_b\rangle$, at proper time τ, are given by

$$\begin{aligned}\mathcal{P}_\tau(K_a, K_b) &= \langle\Psi(\tau)|\Big(\mathcal{O}_a \otimes \mathcal{O}_b\Big)|\Psi(\tau)\rangle \\ &= e^{-2\gamma\tau}|N|^2 \Big|\langle K_S|K_a\rangle\,\langle K_L|K_b\rangle \\ &\quad - \langle K_S|K_b\rangle\,\langle K_L|K_a\rangle\Big|^2 . \end{aligned} \quad (3.1)$$

In the above expression, the projectors operators $\mathcal{O}_{a,b}$ are connected with the kaon states $|K_{a,b}\rangle$ as in (2.6); in a similar way one can construct the probabilities $\mathcal{P}_\tau(K_a, K_b^\perp)$, $\mathcal{P}_\tau(K_a^\perp, K_b)$ and $\mathcal{P}_\tau(K_a^\perp, K_b^\perp)$, by using also the orthogonal states $|K_{a,b}^\perp\rangle$ and the corresponding projectors $\mathcal{O}_{a,b}^\perp$ (cfr. (2.8)). Furthermore, by replacing $\mathcal{O}_a$ (or $\mathcal{O}_b$) in (3.1) with a unit matrix, one obtains the probability $\mathcal{P}_\tau(*, K_b)$ (respectively, $\mathcal{P}_\tau(K_a, *)$) that one of the two kaons decays into the final state f_b (f_a), the other being undecayed; then, $\mathcal{P}_\tau(*, K_b) = \langle\Psi(\tau)|1 \otimes \mathcal{O}_b|\Psi(\tau)\rangle = \mathcal{P}_\tau(K_c, K_b) + \mathcal{P}_\tau(K_c^\perp, K_b)$, for any kaon state $|K_c\rangle$.

At each proper time τ, one has a situation which is formally analogous to the one appearing in standard formulations of Bell's inequalities using spins.[1, 2, 19, 20] That is, $|\Psi(\tau)\rangle$ plays the role of the singlet state of two spin 1/2 particles emitted by a source and the projectors, $\mathcal{O}_{a,b}$, $\mathcal{O}_{a,b}^\perp$, are the analog of spin-polarization operators. The main difference with respect to the standard Einstein-Poldosky-Rosen context is that, while in the spin case the directions along which to measure the spin-polarization are freely chosen by the experimenter, in the neutral kaon case only “polarization” directions that identify specific decay channels are actually allowed. Nevertheless, this restriction does not result in a serious limitation; indeed, as explained in the Appendix, using regeneration techniques, many final “polarizations” states may be experimentally reachable.

In any hidden-variable extension of quantum mechanics, the description embodied in the state $|\Psi(\tau)\rangle$ in (2.19) is completed with additional parameters λ assigning probabilities $p_\lambda^\tau(K_a, K_b)$ to the double-decays into final states f_a and f_b. Also, λ fixes the probabilities $p_\lambda^\tau(K_a, *)$ and $p_\lambda^\tau(*, K_b)$ associated with single decays at time τ of one of the two kaons, the other one being undecayed. The additional parameters generally constitute a statistical ensemble described by a suitable distribution $\rho(\lambda)$ such that $\int {\rm d}\lambda\ \rho(\lambda) = 1$. Then, one asks that the quantum mechanical probabilities (3.1) be reproduced by integration over λ; for instance:

$$\mathcal{P}_\tau(K_a, K_b) = \int {\rm d}\lambda\ \rho(\lambda)\ p_\lambda^\tau(K_a, K_b) , \quad (3.2)$$

while similar relations hold for $\mathcal{P}_\tau(*, K_b)$ and $\mathcal{P}_\tau(K_a, *)$. As a consequence, because of the singlet-like character of $|\Psi(\tau)\rangle$, in any hidden-variable theory the condition $\mathcal{P}_\tau(K_a, K_a) = 0$ should also be satisfied.

Following standard arguments, one also assumes that the probabilities $p_\lambda^\tau(K_a, K_b)$ fulfill the following Bell's locality request

$$p_\lambda^\tau(K_a, K_b) = p_\lambda^\tau(K_a, *)\, p_\lambda^\tau(*, K_b) . \quad (3.3)$$

At any fixed proper time τ, equality (3.3) amounts to standard Bell's locality.[1, 2, 19, 20] In terms of spins, it means that space-like separated polarization measurements cannot influence each other so that the two events have independent statistics. In the case of neutral kaons, due to explicit time-dependence and instability, one might give up the factorization property in (3.3) without the need of superluminal transmission of information at the level of the additional parameters. It is sufficient to imagine hidden-variable theories that predetermine the statistics of future decay events at the moment of the ϕ-meson decay. In the following we shall not consider such theories, but rather assume that kaon decays are local and stochastically independent events, whence (3.3). This assumption can be put to experimental test using regeneration techniques; therefore, hidden-variable theories violating it turn out to be quite unnatural (see the discussion in the Appendix).

Consequences of the assumption (3.3) can be derived using standard techniques;[19] in particular, introducing three different final decay states f_a, f_b and f_c, associated to the kaon states $|K_a\rangle$, $|K_b\rangle$ and $|K_c\rangle$, one can prove that the following inequality must hold:[15]

$$\Big|\mathcal{P}_\tau(K_a, K_b) - \mathcal{P}_\tau(K_a, K_c)\Big| \leq \mathcal{P}_\tau(K_c, K_b) . \qquad (3.4)$$

At any fixed proper time τ, the derivation of (3.4) follows the one in [26] for the time-independent case. However, the same class of inequalities can be deduced from the larger class of Bell's like inequalities which has been obtained in [10] via an argument which essentially adapts the derivation of Wigner, Belinfante and Holt (see [20], Sect. 3.7).

Let us consider for a moment the standard situation based on photon-polarization measures, where time plays no role. Despite its simplicity, when confronting with actual experiments involving coincidence countings, it is not inequality (3.4) which is tested. Due to lack of control of the number of (photon) pairs produced that actually impinge on the detectors, one is generally forced to use more general inequalities, that take into account all losses and nonidealities of the experimental apparatus.[19, 27]

The situation is different in the neutral kaon context; there, one can actually use an inequality like (3.4) without further assumptions. Indeed, tests of an inequality of the form (3.4) can be performed by measuring the phenomenological parameter ε' in (2.14), (2.15) and not directly the probabilities appearing in (3.4). Indeed, by choosing $K_a = K_{\ell+}$, $K_b = K_{00}$ and $K_c = K_{+-}$, an expansion to leading orders in CP and CPT-violating parameters allows us to express (3.4) in terms of the difference $r_{+-} - r_{00}$, or equivalently, using (2.14), in terms of the constant ε'. Then, the inequality to be fulfilled by any local hidden-variable theory accounting for neutral kaon phenomenology and satisfying equality (3.3) reads:

$$\big|\mathcal{R}e(\varepsilon')\big| \leq 3\,|\varepsilon'|^2 . \qquad (3.5)$$

Typically, CP-violations in decay processes are ignored for sake of simplicity in almost all discussions of Bell's inequalities concerning neutral kaons (for an exception see [14]). This means $r_{+-} = r_{00} = 0$ and in such a case (3.5) reduces to a trivial identity. Indeed, setting $r_{+-} = r_{00} = 0$, the projectors in (2.13) both coincide with the projector onto the kaon state $|K_1\rangle$, the decay of $|K_2\rangle$ into a two-pion final state being forbidden. With these simplifying assumptions, there is an identification of the kaon states with definite strangeness or CP-quantum numbers via their decay products. However, in this way, one of the polarization-like direction would be lost. Instead, besides the strangeness eigenstates $|K^0\rangle$, $|\overline{K^0}\rangle$, and the CP-eigenstates $|K_1\rangle$, $|K_2\rangle$, a third orthonormal basis of kaon states $|\widetilde{K}_S\rangle$, $|\widetilde{K}_L\rangle$ is needed to make the typical Bell's argument run. In [10], kaon regeneration methods have been proposed to supply it. The idea of using slabs of regenerating materials in asymmetric ϕ-factories has been put forward in [12] and reconsidered for other purposes in [13]. In the approach of [15] there is no need to consider regeneration as a tool to provide a triple of polarization-like directions K_a, K_b and K_c. Moreover, it is the very fact that small CP-violating effects are not neglected which allows for a connection between direct CP-violations and violations of Bell's locality.

The parameter ε' is accessible to experiments and any experimental determination of it is in principle able to disprove (3.5) and thus (3.4). In the next paragraph we consider the results of the KTeV and the NA48 Collaborations which used uncorrelated kaons.[21, 22] However, the presence and role of ε' could in principle be tested via measuring correlations, that is by determining probabilities of joint events as those appearing in (3.4). In fact, as already explained, when the kaon states $|K_{a,b}\rangle$ are associated to actually occurring decays, then the probabilities $\mathcal{P}_\tau(K_a, K_b)$ in (3.1) are not only formally well-defined, but also measurable quantities.

Finally, let us point out that a violation of (3.5) does not provide a test of the theoretical framework in which the parameter ε' can be estimated.[16-18] It only says that any local hidden-variable theory accounting for stochastic independence of kaon decays and reproducing kaon phenomenology and thus the right value of ε' must, at the same time, fulfill inequality (3.4). Therefore, that very same theory cannot be compatible with a value of ε' which violates (3.5). However, as any test, also the one based on the inequality (3.5) is significant only when giving a negative result, saying that an inequality of the form (3.4) is violated. Only in this case, one is able to experimentally exclude a large set of local deterministic extensions of quantum mechanics. If the experimental data had turned out to be compatible with a vanishing value for ε' (result predicted by the so-called superweak phenomenological model [28]), then, an inequality of the form (3.4) would have simply been unsuited to exclude the hidden-variable theories considered in this paper.

4 Experimental results

While the phenomenological parameter ε is very well known and of the order 10^{-3}, [29] the parameter ε' has only recently been determined with sufficient accuracy. With a fixed-target setup that uses uncorrelated kaons, the KTeV and NA48 Collaborations have measured the double ratio of decay rates in (2.15)

$$\frac{|\eta_{+-}|^2}{|\eta_{00}|^2} \simeq 1 + 6\,\mathcal{R}e\left(\frac{\varepsilon'}{\varepsilon}\right) . \qquad (4.1)$$

Their results are $\mathcal{R}e\,(\varepsilon'/\varepsilon) = (2.80 \pm 0.41) \times 10^{-3}$,[21] and $\mathcal{R}e\,(\varepsilon'/\varepsilon) = (1.85 \pm 0.73) \times 10^{-3}$,[22] which are in rough agreement with the theoretical predictions.[18] The Standard Model further predicts the phase φ' of ε' to be very close to the phase φ of ε.[17] Assuming then $\mathcal{R}e\,(\varepsilon'/\varepsilon) = |\varepsilon'/\varepsilon|$, these experimental determinations allow us to check inequality (3.5), that can be rewritten as

$$R \equiv \left|\frac{\varepsilon'}{\varepsilon}\right| \left(\frac{\cos\varphi}{|\varepsilon|} - 3\left|\frac{\varepsilon'}{\varepsilon}\right| \right) \leq 0\ . \qquad (4.2)$$

Using the most recent determination for ε and the two previously quoted experimental results, one finds

$$R_{\rm KTeV} = 0.89 \pm 0.13\ , \qquad (4.3a)$$
$$R_{\rm NA48} = 0.59 \pm 0.23\ . \qquad (4.3b)$$

The inequality (3.5) is therefore violated by a few standard deviations. The accuracy of the determination of $\mathcal{R}e(\varepsilon'/\varepsilon)$ by the two collaborations will be further improved when all the experimental data are elaborated (the figures previously quoted refer to the study of only about 20% (in the case of KTeV) and 15% (for NA48) of the collected data). Correspondingly, the test on the violation of the inequality (3.5) will also improve; accuracy of about twenty standard deviations can easily be expected. In this way, the test of Bell's locality using neutral kaons will turn out to be at the same level of accuracy of the best tests performed with photon cascades.[30]

We point out that another kaon experiment, KLOE in Frascati, is presently collecting data and its results on the value of ε'/ε will be announced in the near future. This experiment is particularly interesting for our considerations; it takes place at the Daphne ϕ-factory and therefore uses correlated kaons. Thanks to the machine high luminosity, both $\mathcal{R}e(\varepsilon'/\varepsilon)$ and $\mathcal{I}m(\varepsilon'/\varepsilon)$ can be measured independently,[31] so that the working assumption $\varphi' = \varphi$ used above will no longer be necessary. Further, as stressed before, the KLOE setup can also perform a direct check of inequality (3.5) by actually measuring the various probabilities involved. However, due to efficiency limitations[1] (see the discussion in [10]) the accuracy of such a test is expected to be much worse than the one using the measure of ε'.

5 Discussion

As already mentioned, for experimental reasons, the class of inequalities holding in local hidden-variable theories that are tested are in general more complicated than (3.4).[20] One may say that (3.4) stays on the same level as the inequality originally provided by Bell [1] to support the incompatibility of local hidden-variable theories with quantum mechanics. In the case of neutral kaons, a time-dependent version of it reads:

$$\Big| E_\tau(K_a, K_b) - E_\tau(K_a, K_c) \Big| \leq 1 + E_\tau(K_c, K_b)\ , \qquad (5.1)$$

where the correlation functions can be expressed as

$$E_\tau(K_a, K_b) \equiv \mathcal{P}_\tau(K_a, K_b) + \mathcal{P}_\tau(K_a^\perp, K_b^\perp) - \mathcal{P}_\tau(K_a^\perp, K_b) - \mathcal{P}_\tau(K_a, K_b^\perp)\ . \qquad (5.2)$$

When $\tau = 0$, the inequality (5.1) is the standard one and can be clearly contradicted by quantum mechanics. None the less, it has not been submitted to any test: due to actual experimental inefficiencies, one cannot count on perfect anticorrelation among pairs in a singlet state as predicted by quantum mechanics and therefore more complicated inequalities are in general needed.[20] Furthermore, for $\tau \neq 0$, the factorized common exponential damping factor $\exp(-2\gamma\tau)$ in (3.1) compared with the constant 1 on the right hand side of (5.1), confines the possibility of violations to too short times after the ϕ-meson decay.[2]

Like (5.1), inequalities of the type (3.4), if testable through coincidence counting only, would also be plagued by all possible kind of apparatus inefficiencies. Nevertheless, standard neutral kaon phenomenology allows to pass from (3.4) to (3.5) and, therefore, to a condition on the parameter ε'. Since it pertains to the kaon physics, it must be reproduced by any theory aiming to replace standard quantum mechanics neutral kaon phenomenology. Then, the original inequality can be tested by determining ε': no further assumptions are necessary and the corresponding loopholes are avoided.

Let us stress that the violation of (3.5) comes about because local hidden-variable theories that reproduce neutral kaon phenomenology and account for the stochastic independence of neutral kaon decays, become self-contradictory. In fact, they must reproduce a physically measurable parameter ε' whose value happens to violate certain inequalities which must also be satisfied by the same theories. Therefore, beside its role in showing the direct violation of the CP-symmetry, the determination of ε'/ε at the same time provides evidence against a whole class of local completions of quantum mechanics by showing an internal inconsistency of these theories.

Moreover, the result is the first obtained in a subnuclear system and without correlation measurements. Notice, however, that the inequality (3.4) involves joint kaon decays and, as discussed, no apriori obstruction exists to testing it by counting simultaneous decays into two-pion and semileptonic final states. Nevertheless, these measures must eventually reproduce the figure of ε' obtained by any other experiment and cannot show a confirmation of (3.5).

Obviously, not all local hidden-variable theories can be excluded by a determination of ε', but only those for which the inequality (3.4) can be derived. For this, two requirements need to be satisfied: *a*) the truth of the quantum mechanical phenomenological description of neutral kaons and thus the impossibility of observing simultaneous decays of singlet-like entangled kaons into the same final states; and *b*) that the underlying local hidden-variable

[1] For these reasons, the actual determination of ε' at a ϕ-factory involves measures of observables that differ from those that enter the probabilities in (3.4).

[2] This kind of exponential suppression of violation of Bell's inequalities also appears in the analysis of [7-9], where, however, correlations resulting in having or not having a $\overline{K^0}$ at different proper times are used.

theories do not predetermine *ab initio* future decay events which are thus stochastically independent.

We remark that request *a*) is just the acceptance of kaon phenomenology (ultimately of the Standard Model) as a correct description of reality. Instead, request *b*) apriori eliminates those local hidden-variable theories where the probabilities of certain decays are fixed at the moment of the ϕ-meson decay (models have been proposed in [3-7]). Concretely, if we abandoned request *b*) above, then two-pion and semileptonic decays considered in this paper might be correlated *ab initio* and we would not be allowed to use the factorization (3.3). In the standard situation, the experimenter circumvents the possibility that experimental outcomes be predetermined by freely choosing which polarization to measure, without entangled photons "knowing" it. Instead, in the neutral kaon case this is not possible and must be excluded apriori.

Clearly, having put constraints on the class of local hidden-variable theories to be tested, the question is how large is the class which fulfills them. In other words, one may ask how restrictive is request *b*) or how feasible is a hidden-variable theory which reproduces kaon phenomenology and predetermines the statistics of future decay events. In the Appendix we argue that kaon regeneration phenomena [10, 12] may be used to put most severe constraints on them. Indeed, it is shown that the probabilities in (3.4) might be measured not by the statistics of double-decay events at time τ, but rather at later different times $\tau+\tau_a$ and $\tau+\tau_b$ after interposition of regenerating material in one or both the flight paths of the two kaons. The time τ can be chosen such that the presence or not of a slab of regenerating material cannot be known at the moment of the ϕ-meson decay. Furthermore, a lot of freedom is left to the experimenter, since more than one slab of regeneration material can be inserted across the kaons path. Such hidden variable theories, where the decay statistics is preestablished, should also account for all these possible interactions of neutral kaons with the regenerators: they clearly turn out to be very *had hoc* and unviable.

Appendix

Because of their different strangeness quantum number and their strong interactions with nucleons, the neutral kaons $|K^0\rangle$ and $|\overline{K^0}\rangle$ acquire different phases while passing through slabs of materials as copper, lead or carbon.[10] This leads a long-lived kaon $|K_L\rangle$ entering the slab to end up with a (regenerated) short-lived component $|K_S\rangle$ after its exit. Their time-evolution in matter is thus different from the one in vacuum which is described by the hamiltonian $H_{\rm eff} = M - i\Gamma/2$.

A slab of homogeneous regenerating material can be described by a regeneration parameter

$$\rho = \frac{\pi\nu}{m_K}\frac{f-\overline{f}}{\lambda_S-\lambda_L}\ , \tag{A.1}$$

where ν is the number of scattering centers per unit volume, m_K the kaon mass, f, $\overline{f}$ the forward scattering amplitudes for $|K^0\rangle$ and $|\overline{K^0}\rangle$ on nucleons and $\lambda_{S,L}$ the eigenvalues of the vacuum effective hamiltonian. The time-evolution of neutral kaons in matter can be effectively described by an effective hamiltonian $H'_{\rm eff}$; in the $|K^0\rangle$, $|\overline{K^0}\rangle$ basis it reads

$$H'_{\rm eff} = H_{\rm eff} - \frac{2\pi\nu}{m_K}\begin{pmatrix} f & 0 \\ 0 & \overline{f}\end{pmatrix}\ . \tag{A.2}$$

For slabs of materials of sufficiently small thickness d, the time-evolution of the $H_{\rm eff}$-eigenstates $|K_{S,L}\rangle$ can be approximated by [10]

$$e^{-iH'_{\rm eff}\Delta\tau'}|K_S\rangle = |K_S\rangle \ -\ \xi\,|K_L\rangle \tag{A.3a}$$

$$e^{-iH'_{\rm eff}\Delta\tau'}|K_L\rangle = |K_L\rangle \ +\ \xi\,|K_S\rangle\ , \tag{A.3b}$$

where

$$\xi = i\frac{\pi\nu}{p_K}(f-\overline{f})\,d\ , \tag{A.4}$$

with p_K the kaon momentum in the laboratory frame, and $\Delta\tau'$ is the time spent by the kaons in the material.

Consider now two kaons generated in a ϕ-meson decay that evolve in vacuum up to equal proper times τ, when they enter two slabs of regenerating materials with regeneration parameters ρ_a, ρ_b. After times $\Delta\tau'_a$ and $\Delta\tau'_b$ they exit and evolve in vacuum up to proper times $\tau+\Delta\tau'_a+\Delta\tau_a$ and $\tau+\Delta\tau'_b+\Delta\tau_b$. Let us set $\tau_a = \Delta\tau'_a+\Delta\tau_a$ and $\tau_b = \Delta\tau'_b+\Delta\tau_b$, then the initial correlated state Ψ in (2.17) evolves into (compare with (2.18))

$$|\Psi'(\tau+\tau_a,\tau+\tau_b)\rangle \equiv \Big(e^{-iH_{\rm eff}\Delta\tau_a}e^{-iH'_{\rm eff}\Delta\tau'_a}e^{-iH_{\rm eff}\tau}\Big) \otimes \Big(e^{-iH_{\rm eff}\Delta\tau_b}e^{-iH'_{\rm eff}\Delta\tau'_b}e^{-iH_{\rm eff}\tau}\Big)\,|\Psi\rangle\ . \tag{A.5}$$

The probability that the kaon states at proper times $\tau+\tau_a$ and $\tau+\tau_b$ be two generic states $|K_{\eta_a}\rangle$ and $|K_{\eta_b}\rangle$, is given by (see (3.1)):

$$\begin{aligned}\mathcal{P}'_{\tau_a,\tau_b}(K_{\eta_a},K_{\eta_b}) &= |\langle\Psi'(\tau+\tau_a,\tau+\tau_b)|\mathcal{O}_{\eta_a} \otimes\mathcal{O}_{\eta_b}|\Psi'(\tau+\tau_a,\tau+\tau_b)\rangle|^2 \\ &= e^{-2\gamma\tau}|N|^2\Big|\langle K_{\eta_a}|W(\tau_a)|K_S\rangle\langle K_{\eta_b}|W(\tau_b)|K_L\rangle \\ &\quad - \langle K_{\eta_a}|W(\tau_a)|K_L\rangle\langle K_{\eta_b}|W(\tau_b)|K_S\rangle\Big|^2\ ,\end{aligned} \tag{A.6}$$

where

$$W(\tau_a) = e^{-iH_{\rm eff}\Delta\tau_a}\ e^{-iH'_{\rm eff}\Delta\tau'_a}\ , \tag{A.7a}$$

$$W(\tau_b) = e^{-iH_{\rm eff}\Delta\tau_b}\ e^{-iH'_{\rm eff}\Delta\tau'_b}\ . \tag{A.7b}$$

Expanding the states $|K_S\rangle$ and $|K_L\rangle$ in $\langle K_{\eta_j}|W(\tau_j)|K_{L,S}\rangle$, $j=a,b$, above, along orthonormal basis $|K_j\rangle$, $|K_j^\perp\rangle$ such that

$$\langle K_{\eta_j}|W(\tau_j)|K_j^\perp\rangle = 0\ , \tag{A.8}$$

one sees that $(A.6)$ becomes

$$\mathcal{P}'_{\tau_a,\tau_b}(K_{\eta_a},K_{\eta_b}) = |\langle K_{\eta_a}|W(\tau_a)|K_a\rangle|^2\,|\langle K_{\eta_b}|W(\tau_b)|K_b\rangle|^2 \times\mathcal{P}_\tau(K_a,K_b)\ , \tag{A.9}$$

with $\mathcal{P}_\tau(K_a, K_b)$ as in (3.1). Therefore, at least in principle, the probabilities appearing in (3.4) can be measured via (A.9) by freely choosing to insert or not a regeneration slab along the kaons path, without the kaons "knowing" it.

The above argument clearly relies on the condition (A.8). In order to show that this condition is actually implementable, we work in the $|K_{1,2}\rangle$ basis, where $|K_{\eta_j}\rangle \propto (1, \eta_j^*)$ and $|K_j^\perp\rangle \propto (r_j, -1)$ as in (2.7) and (2.8). Then, neglecting higher powers in the small parameters $\epsilon_{S,L}$ and ξ, all of order 10^{-3} or smaller, using (A.3), after some algebra equality (A.8) leads to the following expression for the parameter ξ in (A.4):

$$\xi = \frac{(r_j + \epsilon_L + \eta_j r_j \epsilon_S)\, e^{(i\Delta m - \Delta\gamma/2)\Delta\tau_j} - (\eta_j + \eta_j \epsilon_S + \epsilon_L)}{e^{(i\Delta m - \Delta\gamma/2)\Delta\tau_j} + \eta_j r_j}, \tag{A.10}$$

where $\Delta m = m_L - m_S$ and $\Delta\gamma = \gamma_S - \gamma_L \simeq 2\Delta m$.

When $|K_j\rangle = |K_{\ell^+}\rangle$ as in (2.16), $r_j = 1$ and no choice of η_j renders the right hand side of (A.10) compatible with the smallness of ξ. On the other hand, when $|K_j\rangle$ equals $|K_{+-}\rangle$ or $|K_{00}\rangle$, r_j equals r_{+-} or r_{00} in (2.13); choosing $\eta_j = r_{+-}$ or $\eta_j = r_{00}$ and neglecting higher orders in the small parameters, (A.10) further simplifies:

$$\xi = \varepsilon\Big(1 - e^{(1-i)\Delta m \Delta\tau_j}\Big)\ . \tag{A.11}$$

Such an equality is implementable by an appropriately chosen regenerating material.

Thus, there are no apriori obstructions to computing the joint-probabilities $\mathcal{P}_\tau(K_a, K_b)$ in (3.4) by using (A.9) and thus double-decay rates at times $\tau+\tau_a$ and $\tau+\tau_b$; only in the case when one of the kaon states, say $|K_a\rangle$, equals $|K_{\ell^+}\rangle$, corresponding to semileptonic final states, regeneration techniques fail to provide the link given by equation (A.9). In these cases, no regenerating slab is inserted and τ_a is set to zero. Then, the challenge of hidden-variable theories to be able to predetermine the joint-probabilities appearing in (3.4) comes from the fact that we can decide to obtain the latter by freely choosing to insert or not the appropriate regenerating slabs. For instance, in the simplest situation described above, there are sixteen available possibilities: four choices for $\mathcal{P}_\tau(K_{+-}, K_{00})$ and two choices for $\mathcal{P}_\tau(K_{\ell^+}, K_{+-})$ and $\mathcal{P}_\tau(K_{00}, K_{\ell^+})$ that involve semileptonic final states. Furthermore, one is in principle allowed to insert more than one regenerating slab in the beam path of either kaons. In this way, a lot of freedom of choice is left to the experimenter and these hidden-variable theories must then be able to account for it; they must be very special indeed.

References

1. J. Bell, Physics **1** (1969) 480; Speakable and Unspeakable in Quantum Mechanics, (Cambridge University Press, Cambridge, 1987)
2. M. Redhead, Incompleteness, Nonlocality and Realism, (Clarendon Press, Oxford, 1987)
3. F. Selleri, Lett. Nuovo Cim. **36** (1983) 521
4. J. Six, Phys. Lett. A **150** (1990) 243
5. D. Home, F. Selleri, J. Phys. A **24** (1991) L1073
6. P. Privitera, F. Selleri, Phys. Lett. B **286** (1992) 261
7. A. Datta, D. Home, Found. Phys. Lett. **4** (1991) 165
8. G. Ghirardi, R. Grassi, T. Weber, in the Proceedings of the Workshop on Physics and Detectors for DAΦNE, G. Pancheri, ed., (INFN-LNF, Frascati, 1991)
9. G. Ghirardi, R. Grassi, R. Ragazzon, in The DaΦne Physics handbook, Vol. I, L. Maiani, G. Pancheri, N. Paver eds., (INFN, Frascati, 1992)
10. A. Di Domenico, Nucl. Phys. B **450** (1995) 293
11. A. Di Domenico, in Workshop on K Physics, L. Iconomidou-Fayard, ed., (Editions Frontieres, Gif-sur-Yvette (France), 1997)
12. P.H. Eberhard, Nucl. Phys. B **398** (1993) 155
13. A. Bramon, M. Nowakowski, Phys. Rev. Lett. **83** (1999) 1; Phys. Rev. D **60** (1999) 094008
14. F. Uchiyama, Phys. Lett. A **231** (1997) 295
15. F. Benatti, R. Floreanini, Phys. Rev. D **57** (1998) 1332
16. T.D. Lee, C.S. Wu, Ann. Rev. Nucl. Sci. **16** (1966) 511
17. L. Maiani, in The Second DaΦne Physics Handbook, L. Maiani, G. Pancheri, N. Paver, eds., (INFN, Frascati, 1995)
18. For recent reviews, see: S. Bertolini, M. Fabbrichesi, J.O. Eeg, Estimating ε'/ε, `hep-ph/9802405`, Rev. Mod. Phys. **72**, to appear; A. Buras, Theoretical status of ε'/ε, `hep-ph/9908395`, to appear in the Proceedings of Kaon'99, (Chicago University Press, Chicago, 1999); see also the recent textbook: G.C. Branco, L. Lavoura, J.P. Silva, CP violation, (Oxford University Press, Oxford, 1999)
19. J.F. Clauser, M.A. Horne, Phys. Rev. D **10** (1974) 526
20. J.F. Clauser, A. Shimony, Rep. Prog. Phys. **41** (1978) 1881
21. The KTeV Collaboration, Phys. Rev. Lett. **88** (1999) 22
22. The NA48 Collaboration, Phys. Lett. B **465** (1999) 335
23. F. Benatti, R. Floreanini, Phys. Lett. B **401** (1997) 337
24. F. Benatti, R. Floreanini, Nucl. Phys. B **511** (1998) 550
25. R.A. Bertlmann, W. Grimus, B.C. Hiesmayr, Phys. Rev. D **60** (1999) 114032
26. A. Garuccio, Phys. Rev. A **52** (1995) 2535
27. P.H. Eberhard, Phys. Rev. A **47** (1993) R747
28. L. Wolfenstein, Phys. Rev. Lett. **13** (1964) 562
29. Particle Data Group, Eur. Phys. J. C **3** (1998) 1
30. G. Weihs, T. Jennewien, C. Simon, H. Weinfurter, A. Zeilinger, Phys. Rev Lett. **81** (1998) 5039
31. G. D'Ambrosio, G. Isidori, A. Pugliese, CP and CPT measurements at DaΦne, in The Second DaΦne Physics Handbook, L. Maiani, G. Pancheri, N. Paver, eds., (INFN, Frascati, 1995)